I. A DOUBLE QUANTUM DOT IN AN OSCILLATING ELECTRIC FIELD

Barbara Terhal

Quantum dots are candidate physical systems for the implementation of quantum logic gates. In this proposal I consider a double quantum dot that represents 1 qubit. I show that by applying an AC field, transitions can be induced between the two stationary states, (we could label them as $|0>$ and $|1>$) of the unperturbed system. Unfortunately the rotation matrix does not induce transitions between $|L>$, the state which corresponds to localization of the electron in the left dot and $|R>$, and thus the system does not yet represent a generic 1-bit quantum logic gate. The system is treated in a somewhat idealized way. No temperature effects are taken into account and the double dots are modeled as a single electronproblem in a double well potential.

We consider a small double quantum dot (the system is drawn in figure 1) with the relevant parameters taken such that

$$k_BT << E_C \equiv \frac{e^2}{C} \text{ (smearing of charging energy levels small)} \tag{1}$$

for the level spacing ΔE

$$k_BT << \Delta E \text{ (quantum regime)} \tag{2}$$

$$\Delta E << E_C \tag{3}$$

The tunnel resistance of the barriers that connect the double dot to the leads is very high such that the electron is effectively confined in the double dot and no other electrons can tunnel on the dot. The tunnel resistance of the barrier in between the dots is lower, its height will be modeled in the calculation. The AC source oscillates at high frequency f (microwave) . We are in the photon assisted tunneling regime:

$$hf >> kT \text{ (quantum)} \tag{4}$$

and

$$f \sim \Gamma \tag{5}$$

where Γ is the tunneling rate over the barrier between the dots. We take 2 identical dots. In [2] a similar problem is considered (resonant photon-assisted tunneling through a double quantum dot), but calculations are done with the formalism of second quantization and photon-assisted tunneling is considered between all the available energystates for the electron, not just the groundstates. The confinement potential of each of the dots (labeled with $i = 1, 2$) is modeled by a 2D harmonic potential

$$H_i(\vec{r}) = \frac{\vec{p}^2}{2m} + \frac{1}{2}m\omega_0^2\vec{r}^2 \tag{6}$$

with m the effective mass of the electron. The energy spectrum for a single dot is

$$E_{n_x n_y} = \hbar\omega_0(n_x + n_y - 1) \;\; n_x, n_y = 1, 2, \ldots \tag{7}$$

In the x-direction the two dots are coupled by a barrier of height B. We assume that this does not affect the energyspectrum of the modes in the y direction, so that we consider a 1D problem. In [1] the problem of the 1D double oscillator is discussed. Due to the finite tunnel barrier, the eigen functions with the lowest energy in the double well are non-degenerate. They can be represented as

$$|\phi_a> = \frac{1}{\sqrt{2}}(|L> + |R>) \tag{8}$$

$$|\phi_b> = \frac{1}{\sqrt{2}}(|L> - |R>) \tag{9}$$

with energy difference $E_b - E_a = \hbar\omega'$. State $|L>$ ($|R>$) corresponds to the ground state eigen functions of the uncoupled left (right) dot. State $|\phi_a>$ is even under reflection symmetry $x \leftrightarrow -x$ and $|\phi_b>$ is odd. Assuming that

$$B >> E_C \tag{10}$$

the frequency associated with the level splitting is

$$\omega' = 2\omega_0\sqrt{\frac{2B}{\hbar\omega_0\pi}}\exp\left(-\frac{2B}{\hbar\omega_0}\right) \tag{11}$$

The $|\phi_a>$ and $|\phi_b>$ are stationary states of the system. However, if we start at $t = 0$ with the electron on, say, the left dot $|L>$, after some time t the amplitudes will change into

$$c_L(t) = \exp\left(-\frac{iE_0t}{\hbar}\right)\cos(\omega' t/2) \tag{12}$$

$$c_R(t) = i\exp\left(-\frac{iE_0t}{\hbar}\right)\sin(\omega' t/2) \tag{13}$$

in which $E_0 = \hbar\omega_0$. We now require that ω' is very small, which is equivalent with saying that the tunnel barrier B is very high, such that the timescale τ on which the electron is transferred to the right dot is much larger than the relevant experimental/computational time scales.

A. Effect of an AC source

To the Hamiltonian is added a time-dependent *perturbation*

$$H = H_0 + (-1)^i V\cos(\omega t) \tag{14}$$

with $i = 1$ ($i = 2$) for the left (right) dot. The amplitudes c_a and c_b for the stationary eigenstates become

$$c_a(t) = \exp\left(-\frac{iE_0t}{\hbar}\right)\exp\left(i\omega' t/2\right)\gamma_a(t) \tag{15}$$

$$c_b(t) = \exp\left(-\frac{iE_0t}{\hbar}\right)\exp\left(-i\omega' t/2\right)\gamma_b(t) \tag{16}$$

with the γ's obeying the equations

$$i\hbar\frac{d\gamma_a}{dt} = V\cos(\omega t)\exp\left(-i\omega' t\right)\gamma_b(t)$$

$$i\hbar\frac{d\gamma_b}{dt} = V\cos(\omega t)\exp\left(i\omega' t\right)\gamma_a(t) \tag{17}$$

If we assume that the amplitude of the oscillating field V is small so that we can omit the terms that oscillate with frequency $\omega' + \omega$ in equation (17), the equations can be solved at resonance $\omega = \omega'$. The solutions are

$$\gamma_a(t) = id\cos\left(\frac{Vt}{2\hbar}\right) - ie\sin\left(\frac{Vt}{2\hbar}\right) \tag{18}$$

$$\gamma_b = e\cos\left(\frac{Vt}{2\hbar}\right) + d\sin\left(\frac{Vt}{2\hbar}\right) \tag{19}$$

with d, e constants determined by the initial conditions. If we represent the state $|\phi_a>$ and $|\phi_b>$ as

$$|\phi_a> \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\phi_b> \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{20}$$

qdot.eps

then the time-evolution matrix of the system (up to an overall phase factor) is

$$U(t) = \begin{pmatrix} \exp(i\omega t/2) & 0 \\ 0 & \exp(-i\omega t/2) \end{pmatrix} \begin{pmatrix} \cos(Vt/2\hbar) & -i\sin(Vt/2\hbar) \\ -i\sin(Vt/2\hbar) & \cos(Vt/2\hbar) \end{pmatrix} \tag{21}$$

When at $t = 0$ the system starts in $|\phi_a>$, it will end up in $|\phi_b>$ at time $t = \frac{\hbar\pi}{V}$. When however, at $t = 0$ the system is in state $|L>$, no transitions are induced by the AC field.

---

[1] E.Merzbacher, *Quantummechanics* (Wiley International Edition)
[2] C.A.Stafford, N.S.Wingreen, *preprint no. UGVA-DPT 1995/ 09-901*