Quantum cryptography with Microwaves

by

Pia Döring

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Prof. Dr. David P. DiVincenzo
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1 Introduction

Since several years physicists research the field of quantum information. As applications of quanta like quantum computing become more and more reachable, an interesting field to explore is the field of quantum information transfer. It seems to be a good alternative to today’s technologies due to the properties of photons such as the indivisibility of quanta and the quantum no-cloning theorem.

Particularly nowadays secure communication becomes more important as several spy affairs became public in the last years. The quantum cryptography is a way of encoding information that ensures that only the parties which participate in the exchange of information know the content. Therefore it is advantageous to find a way of using quantum cryptography for information transfer.

In 2007, an experiment with a transfer of optical photons was realized. The detection of optical photons over a distance of 144 km through air was successful [1]. A recent experiment in China shows that quantum communication is even possible over distances of 1200 km. The experiment called QUESS managed to securely transfer photons of the ultraviolet range from a satellite to ground stations which has been reported in June 2017 from several websites such as http://www.sciencecentral.org and http://www.nature.com.

In a paper from 2017, it is stated that the transfer of microwave photons through a fiber can only be managed if the fiber was cooled. Otherwise the disturbance due to thermal photons would become too big to ensure a secure transfer [2]. But the need to cool to milli-Kelvin which was stated in earlier works is abolished and replaced by a need to cool to about 4 K.

Two important aspects which have to be improved and are being discussed in this thesis are the distance over which the transfer is possible and the temperature of the setup. We will find out if and over which lengths the transfer of photons is possible at room temperature.

The frequency range in which we calculate is the microwave range. The number of thermal photons is supposed to decrease for higher frequencies which makes microwave photons very advantageous.
2 Theoretical Principles

In this section I am going to give an overview on the important principles that are needed to understand and describe the difficulties and opportunities of quantum information transfer.

2.1 Quantumcryptography

In this section we will have a look at the encoding of quantum information. It ensures that the transfer of information is secure. If two parties, in the following Alice and Bob, communicate with one another no third party should be allowed to get the information that is sent. To secure this, the photons have to be cryptographied. To achieve a highly secure exchange of information, there are several protocols that allow to get keys for Alice and Bob.

The most common one is the BB84 protocol [3, chapter 12.6.3] where Alice generates \((4 + \delta)n\) with \(\delta \in \mathbb{Z}\) random data bits, string \(a\), to get a key of the length \(n\). She encodes string \(a\) with another random \((4 + \delta)n\)-bit string \(b\) as

\[
\{0\}, \{1\} \text{ if } b = 0 \quad \text{and} \quad \{+\}, \{-\} \text{ if } b = 1
\]

and sends the encoded bits to Bob. He himself generates \((4 + \delta)n\) random data bits \(b'\) and decodes the received bits with \(b'\) in the same manner. Alice announces \(b\) and Bob deletes every bit where \(b \neq b'\). Of the remaining bits he keeps \(2n\). The protocol will be aborted if there remain less than \(2n\) bits because in that case there would not be enough bits to compare and get a \(n\)-bits long key in the end.

Alice and Bob compare the values of \(n\) of the \(2n\) bits. If there are more disagreements than a respectable number, the protocol will be aborted. A high number of disagreements is a sign for too much noise in the transfer channel. If not, Alice and Bob will use the uncompared \(n\) bits as a key.

Another way to get a common key is called the EPR protocol [3, chapter 12.6.3] where Alice and Bob both receive a set of photons from \(n\) entangled pairs. They select a random subset of the photons and test their fidelity e.g. with Bell’s inequality. If they pass the test, Alice and Bob will get a lower bound on fidelity and secret key bits by measuring in jointly set random bases.

2.2 Black Waveguide (Nyquist Theorem)

While Planck’s law for black body radiation is used to describe the ideal form of a black body absorbing and emitting photons of every frequency, we need a formula for the one-dimensional case to calculate the radiation on a transmission line on which the photons are supposed to be transferred. The formula helps us to determine the number of thermal photons which disturb the detection of the signal photons. It was deduced by Nyquist and connects the energy of a photon \(h\nu\) with the voltage \(V\) on the transmission line. The equation can especially be derived in two different ways.
2.2.1 Method of Nyquist

One method given in a paper written by Nyquist himself [4] is using a construction of a circuit consisting of two conductors with resistances $R$ (see fig. 1). The current in the circuit with two resistances in series can then be calculated as $I = \frac{V}{2\pi}$ with the voltage due to the thermal agitation in the conductors $V$. Because of the first law of thermodynamics, it is given that the power transported from I to II equals the power transported from II to I. This especially applies to every chosen frequency range. The reason for this is that if we block all frequencies except from frequencies between $\nu$ and $\nu + d\nu$, the first law of thermodynamics will apply, too.

![Figure 1: Circuits with two resistances $R$](image)

At any time $t$ after the equilibrium the transmission line of length $l$ can be isolated from the conductors, whereby the energy on the waveguide will be conserved and the wave is completely reflected at the ends. The line can then oscillate with frequencies $\nu = n \cdot \frac{v}{2l}$, where $v$ is the velocity of propagation. With the number of modes between $\nu$ and $\nu + d\nu$

$$Nd\nu = \frac{2l}{v}d\nu,$$

we get the total energy within $d\nu$ as

$$Ed\nu = Nd\nu \cdot k_B T = \frac{2l}{v}k_B T d\nu$$

(2)

with the Boltzmann constant $k_B$ and the temperature of the transmission line $T$. Because the power can be calculated with $P = \frac{E}{t} = \frac{E}{Tv}$, the power transferred to the line by each of the two conductors is

$$Pd\nu = \frac{E}{2l/v}d\nu = k_B T d\nu.$$

(3)

This results with $P = I^2R = \left(\frac{V}{2\pi}\right)^2 R = \frac{V^2}{4\pi}$ in the Nyquist theorem

$$V^2d\nu = 4Rk_B T d\nu$$

(4)

respectively

$$V^2 = 4Rk_B T \Delta \nu,$$

(5)
where $\Delta \nu$ is the bandwidth over which the voltage is measured. This is then the formula which describes the voltage in one conductor with resistance $R$ and at temperature $T$.

### 2.2.2 Method of Reif

Reif derives the formula in his book in another way [5, chapter 15.16]. He uses the effective fluctuations of the voltage at a resistance due to random thermal movements of the electrons $V(t)$ and its spectral density $J_+ (\omega)$ which is a measurement for the power of $V(t)$. To calculate this, we picture a resistor connected with a linear amplifier which blocks out frequencies out of the interval between $\omega$ and $\omega + \Delta \omega$.

The mean of the square of the voltage is given by

$$\langle V(t)^2 \rangle = \int_{\omega}^{\omega+\Delta\omega} J_+ (\omega) d\omega. \quad (6)$$

The relation between the resistance and the voltage is constituted by

$$R = \frac{1}{2k_B T} \int_{-\infty}^{\infty} K(s) ds = \frac{1}{2k_B T} \int_{-\infty}^{\infty} \langle V(0)V(s) \rangle_0 ds = \frac{\pi}{2k_B T} J_+(0) \quad (7)$$

where $K(s) = \langle V(0)V(s) \rangle_0$ is the correlation function of $V(t)$.

Because $K(s) = 0$ for $|s| \gg$ correlation time $\tau^*$ and $K(s)$ has a sharp maximum at $s = 0$, the term $e^{-i\omega s}$ in $J_+ (\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} K(s) e^{-i\omega s} ds$ has to equal 1 in the area where $K(s) \neq 0$ if $|\omega\tau^*| \leq 1$.

![Figure 2: Spectral density in dependency on the frequency](image)

This leads to the estimate that $J_+ (\omega) = J_+(0)$ for $|\omega| \ll \frac{1}{\tau^*}$ (see fig. 2) which results in

$$J_+ (\omega) = \frac{2}{\pi} R k_B T \text{ for } |\omega| \ll \frac{1}{\tau^*} \quad (8)$$
By integrating we get the similar formula to equation (5)

\[ \langle V(t)^2 \rangle = \int_{\omega}^{\omega+\Delta\omega} \frac{2}{\pi} R k_B T d\omega = \frac{2}{\pi} R k_B T \Delta \omega = 4 k_B T R \Delta \nu. \]  

(9)

The difference of the two derivations is that in the second derivation in contrast to the first, it is clear that we calculate the mean of the squared voltage and not the squared voltage. The voltage can be an arbitrary function of time.

2.3 Transmission Line Theory

The transmission line theory \([6, \text{chapter 2.1}]\) allows us to calculate the voltage along the line. With the voltage we can later on derive the number of thermal photons. The theory describes a transmission line as a two-wire line. Infinitesimal pieces \(\Delta z\) of the line are modeled as circuits with a resistance \(R\), an inductance \(L\), a conductance \(G\) and a capacitance \(C\). These properties are all given per unit length. Each of them describes a different characteristic of the transmission line. Given in the same order as the properties, the characteristics are self-inductance, close proximity, finite conductivity and dielectric losses between the conductors. The whole transmission line is then modeled by adding infinitesimal circuits in a row.

![Figure 3: Transmission line scheme](image)

With the Kirchhoff laws for current and voltage we get

\[ v(z, t) - R \Delta z i(z, t) - L \Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0 \]  

(10)

\[ i(z, t) - G \Delta z v(z + \Delta z, t) - C \Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0. \]  

(11)

For this circuit we can now assume a time dependency \(v(z, t) = V(z) \cdot e^{i\omega t}\) and
\[ i(z, t) = I(z) \cdot e^{i\omega t} \] which simplifies the equations to

\[ \frac{V(z + \Delta z) - V(z)}{\Delta z} \approx \frac{\partial V(z)}{\partial z} = -(R + i\omega L)I(z) \quad (12) \]

\[ \frac{I(z + \Delta z) - I(z)}{\Delta z} \approx \frac{\partial I(z)}{\partial z} = -(G + i\omega C)V(z) \cdot (13) \]

By differentiating equations (12) and (13) a second time with respect to \( z \) and inserting the original equations we get

\[ \frac{\partial^2 V(z)}{\partial z^2} = -\gamma^2 V(z) \quad (14) \]

\[ \frac{\partial^2 I(z)}{\partial z^2} = -\gamma^2 I(z) \quad (15) \]

with \( \gamma = \sqrt{(R + i\omega L)(G + i\omega C)} \).

### 2.4 Thévenin and Norton Equivalent

Due to the finite temperature of the transmission line the resistance and the conductance cannot be considered to be noiseless. To calculate the noise we can replace the resistance and the conductance by Thévenin or Norton equivalents.

![Figure 4:](image)

For the Thévenin equivalent a resistance \( R \) is replaced by a noiseless resistance \( R' \) and a noisy voltage source \( U(t) \) in series connection so that the tapped voltage does not change [7]:

\[ V = RI = R'I + U(t). \quad (16) \]

The Norton equivalent circuit replaces a resistance \( R \) by a noiseless resistance \( R'' \) and a noisy current source \( I(t) \) in parallel connection so that the tapped current does not change [8]:

\[ i = \frac{V}{R} = \frac{V}{R''} + I(t) \quad (17) \]

The same replacements can be made for the conductance \( G \):

\[ V = \frac{I}{G} = \frac{I}{G''} + U(t) \quad (18) \]

\[ i = GV = G''V + I(t) \quad (19) \]

With those equations we can then replace the resistance and conductance in figure 3 by both either Thévenin or Norton equivalents.
2.5 Detection of Photons

An important part of the quantum transfer is the detection of the sent photons. There are several different possibilities to achieve a proper detection discussed in latest papers. The papers cover different problems of photon detection. For example the problems of thermal photons, photon losses during the transfer and dark counts have to be taken into account in calculations. Dark counts hereby mean that the detector declares a detection despite no signal photon arriving. Every detector has properties that describe how good the problems are compensated. In this section we discuss some of these properties.

One main characteristic of a detector is its efficiency $\eta$. The efficiency describes the probability of actually detecting a received photon. Hence a detector with a low efficiency does not detect the photons reaching it and is therefore not useful. The time interval in which a detector cannot detect two received photons is called the recovery time. It describes the time a detector needs to restore after receiving a photon before it can register another one.

In addition, the detector has to be built in such a way that it can reduce losses and minimize dark counts, so that the probability of registering an actually arriving photon is still high enough. A possibility is to minimize the probability of a dark count in contrast to the probability of a real count [9, App. D].

The fidelity is another important value for the characterization of a detector. It describes how good the state of the sending qubit is transferred to the receiving qubit. If the fidelity is high meaning above $\frac{2}{3}$, the transfer of the state can be seen as quantum.

Also crucial for a detector is the linewidth of the photon which is limited by its generation time. The linewidths of Alice who generates the photon and Bob who detects the photon have to match so that the photon detection is possible. The linewidths also imply the range of thermal photons that has to be included as a disturbing factor in the detection.

Different simulations of detectors in the range of microwaves can be found in several current papers which we will look at in the following subsections. This overall view is just to get to know different realizations and ideas of detectors. Later on we are going to use values from a different paper [9] to describe the probabilities of the detector. These values are typical for modern detectors.

2.5.1 Decoy State Method

The decoy state method was used in an actual experiment [1] and is not just a theoretical consideration of how the detection of photons would work. It took place on the Canary Islands Teneriffe and LaPalma. In the experiment, optical photons were encoded with the BB84 method and sent and received over a length of 144 km.

The physicists did not only send signal photons but they also used decoy photons which they sent between the beams of signal photons. With the help of these decoy photons the detector got to know an upper boundary for the fraction of photons that is tagged by an eavesdropper. This avoids that the laser pulses with the signal photons have to be too heavily attenuated to be attractive for a measurement or that they are too little attenuated so that eavesdropping is still possible.
2.5.2 Quantum Nondemolition Detection

Another group developed a simulation for the detection of photons which does not destroy them during the process [10]. For this simulation they used $N$ transmons which are basically noninteracting artificial atoms. They can be described as a three-level system.

Two fields are used in the simulation, a control field which is the one supposed to be measured and a probe field. They are both close to resonance with the transmons. The control field can excite the first transmon so that it then excites the next which continues through the $N$ transmons. Thus the probe field is disturbed at the end. In the absence of a control field the probe field can pass the system undisturbed. The disturbance can be measured by comparing the frequencies of the probe field before passing the system and afterwards.

2.5.3 Catch and Release

The authors of another paper [11], a group from the University of California, are discussing several coupled elements which are used to catch and shortly store photons before they are then released. For this theoretical simulation a qubit is used to initiate photons which are resonant to a cavity. The frequency of the qubit $f_q$ can be tuned in between 6 and 7 GHz. This then determines the coupling of the qubit and the resonator which has the frequency $f_r = 6.57$ GHz.

The coupling can be completely turned off by setting $f_q = f_r - 400$ MHz$\approx 6.57$ GHz$-$400 MHz.

In addition, there is a coupling $\kappa_C$ between the resonator and the transmission line. It can be varied between zero and maximum coupling $\kappa_{max} = \frac{1}{\tau_s}$. The variation is controlled with a coupler bias whose rising time limits the coupling.

If the coupling is off or weak, the photon would decay in the resonators due to losses. If the coupling is strong enough, the photon would be emitted.

The coupling between the resonator and the transmission line determines the decay time in the resonator. This can be shown by exciting the resonator at a distinct coupling strength $\kappa_C$, setting the coupling to zero for a storage time $\tau_s$ and then back to $\kappa_C$ which leads to a coherent release of photons. The signal decays after the sharp onset with a decay time $T_d \approx \frac{2}{\kappa_C}$.

2.5.4 Harmonic Oscillators as Intermediate Stage

In this part, we will have a look at a paper which discusses the general improvement of the transfer of photons and in particular the improvement of the fidelity [2]. The paper treats the problem that the transfer channel is warmer than the refrigerators in which the qubits are situated and which are at a few milli-Kelvin. Although they only talk about $T \approx 4$ K which is not the temperatures we want to have a look in, it is actually an advantage to the assumption that the channel has to be cooled to mK to minimize the number of thermal photons which was made before.

They consider two qubits of which each has a ground state $|0\rangle$ and an excited state $|1\rangle$ with a difference $\hbar \omega_0$. The qubits are connected via an unidirectional channel. The task would then be to transfer a state $|\Psi\rangle$ which is prepared at the first node to the second node.

The fidelity decreases for temperatures $T_{ch} > \frac{\hbar \omega_0}{\kappa_B}$ because the thermal photons of the channel would then disturb the transfer. Instead of a direct connection
between the qubits and the channel, harmonic oscillators are used as an interme-
diate stage. This is advantageous because their swap operation is independent
of the number of states in the channel. They mention more operations to improve the transfer. But these are not useful
for the subject of this thesis. To get to know the other improvements I would highly recommend to read the paper [2].
3 Calculations and Results

3.1 Solutions for the Transmission Line Theory

The transmission line problem we look at is a transmission line terminated at both ends [6, chapter 2.3]. The transmission line itself is at a temperature $T_{\text{high}}$, while the terminating resistances are at a temperature $T_{\text{low}}$. Over the length $L$ of the transmission line we string together infinitesimal pieces $\Delta z$ of the type shown in figure 3.

$$I(0)$$ at $T_{\text{high}}$  
$$V(0)$$  
$Z_T$ at $T_{\text{low}}$  
$z = 0$

$$I(L)$$ at $T_{\text{low}}$  
$$V(L)$$  
$Z_T$

**Figure 5:** Terminated transmission line

The solutions of the differential equations for the voltage (14) and the current (15) in the transmission line theory can be solved with an ansatz

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}, \quad (20)$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}, \quad (21)$$

where $V_0^+$ and $I_0^+$ are the amplitudes of the waves travelling in $+z$-direction and $\gamma = \alpha + i\beta = \sqrt{(R + i\omega L)(G + i\omega C)}$. The factor $\gamma$ consists of a real part $\alpha$ which describes the attenuation of the wave and the imaginary part $\beta$ which characterizes the oscillation of the voltage respectively the current. The amplitudes correlate which can be derived from equation (12) or equation (13):

$$\frac{\partial V(z)}{\partial z} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -(R + i\omega L)(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}) \quad (22)$$

$$\Leftrightarrow I(z) = \frac{\gamma}{R + i\omega L} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \quad (23)$$

$$\Leftrightarrow I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad (24)$$

with the characteristic impedance $Z_0 = \frac{R + i\omega L}{\gamma} = \sqrt{\frac{R + i\omega L}{G + i\omega C}}$.

By looking at a line terminated on both ends with impedances $Z_T$, we can derive a relation between the amplitudes of the waves in $+z$- and $-z$-direction. For a terminated line at $z = 0$ and $z = L$ we get two conditional equations:
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\[ Z_T = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{Z_0(V_0^+ - V_0^-)} \] (25)

\[ Z_T = \frac{V(L)}{I(L)} = \frac{V_0^+ e^{-\gamma L} + V_0^- e^{\gamma L}}{Z_0(V_0^+ e^{-\gamma L} - V_0^- e^{\gamma L})} \] (26)

The first condition (eq. (25)) yields the relation \( V_0^- = V_0^+ \frac{Z_T - Z_0}{Z_T + Z_0} \) for the amplitudes of the two directions. From equation (26) we extract a term for the length of the transmission line:

\[ Z_T = \frac{V_0^+ e^{-\gamma L} + V_0^- e^{\gamma L}}{Z_0(V_0^+ e^{-\gamma L} - V_0^- e^{\gamma L})} \Leftrightarrow V_0^- = V_0^+ \frac{Z_T - Z_0}{Z_T + Z_0} e^{-2\gamma L} \]

\[ \Leftrightarrow 1 = e^{-2\gamma L} \]

Because of the real and imaginary part of \( \gamma \) this condition states that the length of the transmission line would as well have an imaginary part. This is not suitable and therefore the imaginary part has to equal zero to evaluate the equation. As we have no further interest in the condition, it has just been included for the sake of completeness.

3.2 Noise by Thévenin or Norton Equivalent

If we replace every resistance and conductance by its Thévenin or Norton equivalent, we would have to calculate an inhomogenous differential equation. While the noise source is a random function of the time, the solution would become even more difficult.

Instead of calculating the whole transmission line with inserted noise sources, we take advantage of the information we already got about the dependency of the voltage on \( z \). In the following we therefore only use the exponential dependency \( e^{i\alpha z} \) describing the attenuation of the wave along the transmission line. This is feasible because we will calculate the power a few chapter later and hence we only need the square of the absolute value of the voltage which abolishes the factor \( e^{i\beta z} \). In addition to that, we are only interested in the attenuation of the wave as we want to calculate the attenuation of the power given at equilibrium.

3.3 Distortionless Case

The distortionless transmission line is a special case where the real and the imaginary part of \( \gamma \) have an easy dependence on the frequency of the wave \( \omega \).

In general the dependency of the imaginary part on \( \omega \) is not linear. This leads to a dependency of the velocity on the frequency \( v = \frac{\omega}{\gamma} \). Due to this there is dispersion. The imaginary part of \( \gamma \) is only proportional to \( \omega \) and the velocity therefore independent of the frequency in the case of no loss or in a distortionless transmission line.

The required term for no distortion is

\[ \frac{R}{L} = \frac{G}{C} \] (27)
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For this condition the factor $\gamma$ simplifies as follows

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)} = i\omega\sqrt{LC} \sqrt{\left(1 + \frac{R}{i\omega L}\right) \left(1 + \frac{G}{i\omega C}\right)} \quad (27)$$

$$= i\omega\sqrt{LC} \left(1 + \frac{R}{i\omega L}\right) = R\sqrt{\frac{C}{L}} + i\omega\sqrt{LC} \equiv \sqrt{RG} + i\omega\sqrt{LC} = \alpha + \beta$$

$$= i\omega\sqrt{LC} \left(1 + \frac{R}{i\omega L}\right) = R\sqrt{\frac{C}{L}} + i\omega\sqrt{LC} \equiv \sqrt{RG} + i\omega\sqrt{LC} = \alpha + \beta \quad (28)$$

$$\alpha = \sqrt{RG} \quad \beta = \omega\sqrt{LC} \quad (29)$$

The advantage of the distortionless transmission line is not only that the velocity is independent of the frequency and hence there is no dispersion, but in addition to that the factor $\alpha$ which describes the attenuation of the wave is not explicitly dependent on the frequency as well. Therefore we do not have to consider different attenuations or velocities for different frequencies.

In fact, the resistance $R$ is slightly dependent on the frequency. But with the choice of the attenuation constants in section 3.6 this is not relevant in this thesis and just mentioned to not give false information.

3.4 Transferring Voltage into Number of Photons

For the evaluation of the feasibility of detecting a sent photon despite thermal photons in the transmission line, we need to calculate the power first. Afterwards we deduce the number of thermal photons on the transmission line in similarity to the 3D-case derived by Planck [12]. We will do that for an equilibrium and proceed with the non-equilibrium case with the finite length later on.

The absorbed power on the transmission line in the interval between $\omega$ and $\omega + d\omega$ equals the velocity of the waves $c'$ times the number of waves per unit length $n = \frac{1}{2\pi} \frac{d\omega}{c'}$ times the energy $\epsilon(\omega)$ [5, chapter 15.17]. The factor $c' \cdot n$ can be understood as an inverse time which describes the number of waves arriving per unit time.

$$P(\omega)d\omega = c' \left(\frac{1}{2\pi} \frac{d\omega}{c'}\right) \epsilon(\omega) \quad (30)$$

This results with the mean energy $\epsilon(\omega) = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$ in

$$P(\omega)d\omega = \frac{\hbar\omega}{2\pi} e^{\hbar\omega/k_B T} - 1 d\omega \quad (31)$$

The power and the number of thermal photons along the transmission line are certainly connected. To point this out, we first have a look at the 3D-case
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described by Planck. This derivation is shown in Reif [5, chapter 9.13-9.15].
The number of photons of both polarisations per unit volume with a frequency in between $\omega$ and $\omega + d\omega$ can be derived by adding up the mean number of photons $f(k)$ with a specific value of $k$ over the volume of the $k$-space within a spheric shell with radius $k = \frac{\omega}{c}$ and $k + dk = \frac{\omega + d\omega}{c}$ and multiplying with 2:

$$\bar{N}(\omega)d\omega = 2 \cdot f(k) \cdot 4\pi k^2 dk$$

(32)

where $f(k)dk = \frac{1}{(2\pi)^3} \frac{1}{e^{\hbar \omega / k_B T} - 1} dk$. We now substitute $k = \frac{\omega}{c}$ with $\omega$ and rearrange terms:

$$\bar{N}(\omega)d\omega = 2 \cdot \frac{1}{e^{\hbar \omega / k_B T} - 1} \cdot 4\pi \left( \frac{\omega}{c} \right)^2 d\omega = \frac{4 \pi \hbar \omega^3}{c^3} \cdot \frac{1}{e^{\hbar \omega / k_B T} - 1} d\omega$$

(33)

On the other hand the power can be calculated as follows. F. Reif does this calculation in his book beginning with calculating the power which is absorbed by an ideal black body. Further on Reif uses the detailed equilibrium and integrates over the solid angle. The detailed derivation can be seen in Reif [5, chapter 9.15]. In the end we get:

$$P(\omega)d\omega = \frac{2\pi \hbar \omega^3}{c^3} f(k)d\omega = \frac{2 \pi \hbar \omega^3}{c^3} \cdot \frac{1}{e^{\hbar \omega / k_B T} - 1} d\omega$$

(34)

By comparing the results for the number of photons and the power, we can see the following relation:

$$\bar{N}(\omega)d\omega = \frac{4}{\hbar \omega c^3} \cdot P(\omega)d\omega$$

(35)

In the one-dimensional case we want to look at the flux in the end. We can do the same for the 3D-case by multiplying $\bar{N}$ with the velocity of the waves $c^\prime$.

$$N(\omega)d\omega = c^\prime \cdot \bar{N}(\omega)d\omega = \frac{4}{\hbar \omega} \cdot P(\omega)d\omega$$

(36)

Now we derive the number of photons per unit length and frequency equivalently to the derivation of $\bar{N}$ in the three-dimensional case (eq.(32)). In one dimension we do not need the factor 2 because there is only one direction of polarisation. Hence the number of photons with $k$ which is $f(k) = \frac{1}{2\pi} \frac{1}{e^{\hbar \omega / k_B T} - 1}$ in the one-dimensional issue gives the number of photons per unit length and frequency within $d\omega$

$$\bar{N}(\omega)d\omega = f(k)dk = \frac{1}{2\pi} \frac{1}{e^{\hbar \omega / k_B T} - 1} \frac{d\omega}{c}$$

(37)

As announced we want to look at the number of photons respectively the number of photons per unit time per unit frequency. Therefore we again multiply with $c^\prime$:

$$N(\omega)d\omega = c \cdot \bar{N}(\omega)d\omega = \frac{1}{2\pi} \frac{1}{e^{\hbar \omega / k_B T} - 1} d\omega$$

(38)
This shows that the relation between the number of photons and the power (see eq.(31)) in the 1D-case is given by:

$$N(\omega)d\omega = \frac{1}{\hbar\omega} P(\omega)d\omega$$  \hspace{1cm} (39)

---

### 3.5 Difference in Temperature

Since the resistances which terminate the transmission line are at a lower temperature than the line itself (see fig.5), the number of thermal photons at the ends of the line is different compared to the number of thermal photons along the line. For very low temperatures which are virtually zero there are no thermal photons in the range between $z = 0$ and $z = \epsilon$ respectively $z = L - \epsilon$ and $z = L$.

Assuming that the temperature at the resistances is $T_{\text{low}} = 0.5$ K the number of thermal photons is much smaller than the number of thermal photons in the transmission line which is at a temperature $T_{\text{high}}$ of the order of room temperature. The thermal photons of the colder part are attenuated as well which leads to the assumption that the number of photons out of that area are negligible compared to the thermal photons due to the hotter transmission line. Nevertheless we include the number of photons coming out of the cold area at first to show that the effect of them is very little.

Concerning the spectral density of the voltage we expect an additional contribution up to a cutoff $\frac{1}{\hbar\omega_{\text{low}}}$ which is much smaller than the cutoff of the spectral density at $T_{\text{high}} \frac{1}{\hbar\omega_{\text{high}}}$ (see fig.7). If the frequency we look at lies between the two cutouts, we would not have to consider any impact of the non-zero temperature of the terminating resistances.
3.6 Power and Number of Photons for Finite Length

In the case of an infinite transmission line at one temperature the number of thermal photons stays the same along the line because of its equilibrium state. Therefore the attenuation of the voltage due to losses plays no role. In the regarded case of a finite transmission line which is terminated by resistances at lower temperature than the rest of the line, the equations for the power and the number of photons have to be corrected considering the attenuation.

The exponential attenuation of the voltage given by

\[ V(z) \propto e^{-\alpha z} \quad (40) \]

is the only important dependency of the voltage because for the power the square of the absolute value is considered:

\[ P = |I|^2 R = \frac{|V|^2}{R} \quad (41) \]

Hence the imaginary part of \( \alpha \) is not included in the equation for the power. Because of the square the dependency of the power on \( z \) is

\[ P \propto e^{-2\alpha z}. \quad (42) \]

The maximum power meaning the power of the equilibrium which is attenuated along the transmission line is given by equation (31) with the inserted temperature \( T_{high} \):

\[ P_{eq}(\omega)d\omega = \frac{1}{2\pi} \frac{h\omega}{e^{h\omega/k_B T_{high}} - 1} d\omega \]

The power can then be separated in a contribution \( P_{left} \) which describes the power of the waves going left on the transmission line and a contribution \( P_{right} \) which is defined equivalently. In the equilibrium case the power of each direction is half of the overall power \( P_{eq} \): \( P_{eq, left} = P_{eq, right} = \frac{P_{eq}}{2} \).
At the terminating resistances $Z_T$ the power given by equation (31) with $T = T_{high}$ is virtually zero. Instead there is another, much lower power which produces thermal photons because of the non-zero temperature $T_{low}$. We have therefore an overall power consisting of a part of $P_{T_{high}}(\omega)$ and $P_{T_{low}}(\omega)$. The total power for each direction in dependency on $z$ can be described by

$$P_{left}(z, \omega) = \left(1 - e^{-2\alpha(L-z)}\right)P_{eq,left,T_{high}}(\omega) + e^{-2\alpha(L-z)}P_{eq,left,T_{low}}(\omega)$$

(43)

$$P_{right}(z, \omega) = \left(1 - e^{-2\alpha z}\right)P_{eq,right,T_{high}}(\omega) + e^{-2\alpha z}P_{eq,right,T_{low}}(\omega).$$

(44)

The prefactors of $P_{eq,T_{high}}(\omega)$ ensure that the contribution of the power of $T_{high}$ of the left- respectively right-going wave equals zero at the resistances $Z_T$ at $z = L$ respectively $z = 0$. To evaluate the validity of our equation we want to have a look at the plotted powers meaning equations (43) and (44) and their sum.

To make these plots we first of all need some values for the attenuation of the wave. It consists of attenuations due to several reasons. The constant is a sum of losses due to metal conductivity, dielectrical loss tangent, conductivity of dielectric and radiation [13]. The dielectrical loss tangent is proportional to the frequency. Therefore this loss type becomes more important for higher frequencies e.g. microwave frequencies than e.g. the metal conductivity which is proportional to the square root of the frequency.

The dielectric loss tangent is independent of the geometry and only dependent on the type of dielectric material. The loss constant $\alpha_d$ is proportional to the loss tangent $\tan \delta$. It is a measure of the ratio of the conductance to the susceptance meaning a ratio of the lossy reaction on the electric field to the lossless reaction.

In the following we will only consider dielectric loss tangent loss and leave out the other attenuation as they are negligible for microwave frequencies.

This is not very consistent with the earlier discussed distortionless case because the conductance $G$ is seen as big enough to dominate the other effects. The resistance $R$ is however very low as we do not include losses due to metal conductivity. Hence the condition (27) states that the ratio $\frac{L}{C}$ becomes very little which is not usual.

Nevertheless we keep the exponential attenuation $e^{-\alpha z}$ which is not explicitly dependent on the frequency although the attenuation due to dielectric loss tangent is itself linear in the frequency.

We adapt values for the loss constant $\alpha_d$ from an online website which explains losses on a transmission line. The website contains a graph titled 'TEM media dielectric loss, ER=1' [13] which shows the dependency of the loss constant on the frequency. The values for $\alpha_d$ that are used later on has been calculated with this graph. In the plot the loss is given in $\frac{dB}{cm}$. To get the attenuation constant $\alpha$ that we want to insert in our calculation, we convert the loss constant $\alpha_d$ from the source. For this we use:

$$dB = 10 \cdot \log_{10} \left(\frac{P_1}{P_2}\right) \text{ with } P_{1,2} = e^{-\alpha_{1,2}}$$
The difference between \( z_1 \) and \( z_2 \) is \( \Delta z = 1 \text{ cm} \) which ensures that the value of \( \alpha \) corresponds to \( \alpha_d \) given in \( \text{dB/cm} \). In addition to that we multiply \( \alpha_d \) with the same difference \( \Delta z \) to get a value in dB. Hence the conversion reads:

\[
\alpha_d \cdot \Delta z = 10 \cdot \log_{10} \left( e^{-\alpha (z_1 - z_2)} \right) \tag{45}
\]

\[
\Leftrightarrow 10^{0.1 \cdot \alpha_d \cdot \Delta z} = e^{-\alpha \cdot \Delta z} \tag{46}
\]

\[
\Leftrightarrow \alpha = -\frac{1}{\Delta z} \ln \left( 10^{0.1 \cdot \alpha_d \cdot \Delta z} \right) \tag{47}
\]

The graph gives different values for the attenuation for different tan \( \delta \). As there was no proper information given which values for tan \( \delta \) are feasible, we calculate with the intermediate values for tan \( \delta = 0.005 \). At the end we compare those with the values for tan \( \delta = 0.001 \) which is the lowest value for tan \( \delta \). For tan \( \delta = 0.005 \) the slope is \( -0.5 \text{ dB/cm} \) since the attenuation is at \( -0.5 \text{ dB/cm} \) for \( f = 110 \) GHz.

The frequency we use to have a look at the graphs of the power is \( f = 6 \) GHz which is a very often used frequency in the paper about microwave detectors. With the slope of the plot for tan \( \delta = 0.005 \) the attenuation for \( f = 6 \) GHz is:

\[
\alpha_d = \frac{-0.5 \text{ dB/cm}}{110 \text{GHz}} \cdot 6 \text{ GHz} = -0.027 \text{ dB/cm} \quad \Rightarrow \quad \alpha = 0.63 \text{ m}^{-1}
\]

For the plots we also use a transmission line of the length \( L = 10 \) m. Later on when we try to reduce the number of thermal photons, we change the length of the transmission line. This value is just used to graphically illustrate the behaviour of the power along the transmission line.
3 CALCULATIONS AND RESULTS

Figure 8: Power in dependency on $z$ for $\omega = 2\pi \cdot 6$ GHz, $T_{\text{high}} = 300$ K, $T_{\text{low}} = 0.5$ K, $L = 10$ m, $\alpha = 0.63 \frac{1}{\text{m}}$ with the added power of $T_{\text{low}}$

As the total power $P_{\text{tot}}$ has its maximum at $z = \frac{L}{2}$ and decreases to half of the maximum at $z = 0$ and $z = L$, the assumed descriptions of the left- and right-going power seem right.

We now want to show that the effect of the power initiated by thermal photons due to $T_{\text{low}}$ is negligibly low. We only plot the power consisting of the contribution of $T_{\text{high}}$. 
3 CALCULATIONS AND RESULTS

Figure 9: Power in dependency on $z$ for $\omega = 2\pi \cdot 6 \text{ GHz}$, $T_{\text{high}} = 300 \text{ K}$, $T_{\text{low}} = 0.5 \text{ K}$, $L = 10 \text{ m}$, $\alpha = 0.63 \frac{1}{\text{m}}$ without the added power of $T_{\text{low}}$

Looking at the values of the power, the difference between the plots of figures 8 and 9 seems small which can be pointed out by plotting the power with the added contribution of $T_{\text{low}}$ and the power without its contribution in one graph. For better comparison we zoom in at the crossing point of $P_{\text{left}}$ and $P_{\text{right}}$. 
3 CALCULATIONS AND RESULTS

Figure 10: Power in dependency on $z$ for $\omega = 2\pi \cdot 6$ GHz, $T_{high} = 300$ K, $T_{low} = 0.5$ K, $L = 10$ m, $\alpha = 0.63\frac{1}{m}$ to point out the effect of the power of $T_{low}$

The relative difference between the powers is of the order $10^{-4}$ which proves that the part of $T_{low}$ is negligible. Therefore we use

$$P_{left}(z, \omega) = \left(1 - e^{-2\alpha(L-z)}\right) P_{eq.,left,T_{high}}(\omega)$$ (48)
$$P_{right}(z, \omega) = \left(1 - e^{-2\alpha z}\right) P_{eq.,right,T_{high}}(\omega)$$ (49)

in the following calculations.

Out of these equations we can derive the number photons with equation (39):

$$N_{left}(z, \omega) = \frac{1}{\hbar \omega} P_{left}(z, \omega) = \frac{1}{\hbar \omega} \left(1 - e^{-2\alpha(L-z)}\right) P_{eq.,left,T_{high}}(\omega)$$
$$= \frac{1}{\hbar \omega} \left(1 - e^{-2\alpha(L-z)}\right) \left\{\frac{1}{2\pi} e^{\hbar \omega / k_B T_{high}} - 1\right\}$$
$$= \left(1 - e^{-2\alpha z}\right) \frac{1}{4\pi} e^{\hbar \omega / k_B T_{high}} - 1$$
$$\frac{1}{\hbar \omega} N(\omega) = N_{left}(\omega)$$

and equivalently:

$$N_{right}(z, \omega) = \left(1 - e^{-2\alpha z}\right) \frac{1}{4\pi} e^{\hbar \omega / k_B T_{high}} - 1$$
$$\frac{1}{\hbar \omega} N(\omega) = N_{right}(\omega)$$
3 CALCULATIONS AND RESULTS

Hence the total number of photons along the transmission line can be calculated with:

\[ N_{\text{tot}}(z, \omega) = \left( 1 - \frac{1}{2} \left( e^{-2\alpha(L-z)} + e^{-2\alpha z} \right) \right) \frac{1}{2\pi} e^{\hbar\omega/k_B T} = 1 \]  

\[ (50) \]

Figure 11: Number of photons in dependency on \( z \) for \( \omega = 2\pi \cdot 6 \text{ GHz}, T_{\text{high}} = 300 \ K, T_{\text{low}} = 0.5 \ K, L = 10 \text{ m}, \alpha = 0.63 \frac{1}{\text{m}} \)

3.7 Detector Values

To find out if the detection of sent photons is possible despite the thermal photons on the transmission line, we have to compare the number of both. In regard to that we can derive the number of photons per second for both kinds. The number of signal photons can be calculated with some detector values. We extract these values from a paper which approaches the detection of photons as well [9]. The values can be used because they are pretty much alike for all detectors. First of all we note that the generation of a photon takes a finite time \( \tau_{\text{gen}} = 254 \text{ ns} \) [9]. This leads to a number of generated photons per second of \( n = 3.94 \cdot 10^6 \frac{1}{s} \).

Due to losses during the transfer of the photons along the transmission line only a fraction of those generated photons reaches the detector. We now assume a transmission line which ensures the transfer of each generated photon. This is of course not feasible for application but an efficiency which is lower than 1 would
not change the order of the number of photons. Another later on used value is the linewidth of the frequency. The generated photons do not have the exact same frequencies. But the generated frequency always lies inside a range around the desired frequency. This range is \( \delta \omega \approx 2\pi \cdot 1 \text{ MHz} \) [9].

### 3.8 Results

The number of thermal photons disturbing the process can be derived with equation (50). Because the frequency of a signal photon has a finite linewidth we need to integrate the number of thermal photons over the range of the linewidth and not only insert the frequency of the photon in the formula. The likely regarded frequency lies in the X-band which is a band of frequencies in the microwave radio region. It is \( \omega = 2\pi \cdot 6 \text{ GHz} \). The linewidth is at the magnitude of \( \delta \omega \approx 2\pi \cdot 1 \text{ MHz} \) [9]. Hence we calculate the number of thermal photons for the infinite transmission line as follows:

\[
N = \frac{1}{2\pi} \int_{-\frac{\Delta \omega}{2}}^{\frac{\Delta \omega}{2}} \frac{1}{e^{\frac{\lambda \omega}{k_B T}} - 1} d\omega \approx 1.65 \cdot 10^8 \frac{1}{s}
\]

To find out how many thermal photons arrive at the detector in the same time that one signal photons arrives, we multiply the number of thermal photons per second with the time that it takes to generate a photon \( \tau_{\text{gen}} = 254 \text{ ns} \) [9]:

\[
N_{\text{thermal}} = 254 \text{ ns} \cdot N \approx 41.97
\]

As this is the number of thermal photons if the transmission line was on an equilibrium it decreases at the ends of a finite transmission line as derived in equation (50):

\[
N_{\text{thermal}}(z) = 41.97 \cdot \left(1 - \frac{1}{2} \left(e^{-2\alpha(L-z)} + e^{-2\alpha z}\right)\right)
\]

To now calculate the number of thermal photons at the detector meaning at the end of the transmission line, we use the value for the attenuation constant \( \alpha \) for \( \tan \delta = 0.005 \) we derived in equation (47). With the earlier used length of \( L = 10 \text{ m} \) the shape of the number of thermal photons looks as shown in figure 12.
Despite the attenuation the number of thermal photons arriving at the detector meaning at \( z = L \) in the same time that one signal photon arrives is approximately 21. This is too high to detect the signal photon. Hence we try to improve the detection and get \( N_{\text{thermal}}(z = L) \) under 1 by trying other frequencies, lengths and temperatures. At first we vary the frequency staying at \( T_{\text{high}} = 300 \text{ K} \) and try 1 GHz and 30 GHz comparing the results with the values for 6 GHz. The chosen frequency of 30 GHz is used because frequencies above 30 GHz are more difficult to generate. We do this for the lengths \( L = 0.1 \text{ m}, 1 \text{ m}, 10 \text{ m} \). The values for \( N_{\text{thermal}} \) and \( N_{\text{thermal}}(z = L) \) are calculated by integrating over the frequency with the linewidth and by adding the \( z \)-dependency.
The number of thermal photons at the detector does not go under 1 for the tested lengths. Although it is lower for higher frequencies which is a result of the higher attenuation and lower number of thermal photons at that frequency. The number of thermal photons decreases rapidly for the both lower frequencies as we shorten the transmission line. This development does not continue this strong for even shorter lengths. The reason for the intense decreasing is that the left- respectively right-going power has to equal zero on the right respectively left end of the transmission line. Therefore the number in the middle is not at \( N_{\text{thermal}} \) because its distance to the cold termination is too small. At even shorter lengths the ‘slope’ flattens.

For a frequency of 30 GHz the number of thermal photons approaches 0.1 for a length of about 4 mm. But as we would like to be able to transfer photons over a longer distance, we now vary the temperature of the transmission line for a length of 1 m and the three frequencies we inserted before.

### Table 1: Values for different lengths \( L, T_{\text{high}} = 300 \text{ K}, \tan \delta = 0.005 \)

<table>
<thead>
<tr>
<th>( f = \frac{n}{2\pi} ) [GHz]</th>
<th>( \alpha \left[ \frac{1}{m} \right] )</th>
<th>( L ) [m]</th>
<th>( N_{\text{thermal}} )</th>
<th>( N_{\text{thermal}}(z = L) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>1</td>
<td>252.46</td>
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<td></td>
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<td>1.93</td>
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<tr>
<td></td>
<td></td>
<td>10</td>
<td>4.15</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Values for different temperatures \( T_{\text{high}}, L = 1 \text{ m}, \tan \delta = 0.005 \)

<table>
<thead>
<tr>
<th>( f = \frac{n}{2\pi} ) [GHz]</th>
<th>( \alpha \left[ \frac{1}{m} \right] )</th>
<th>( T_{\text{high}} ) [K]</th>
<th>( N_{\text{thermal}} )</th>
<th>( N_{\text{thermal}}(z = L) )</th>
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<td>300</td>
<td>8.29</td>
<td>4.14</td>
</tr>
</tbody>
</table>
3 CALCULATIONS AND RESULTS

Only for a temperature of 50 K we get a number of thermal photons at the detector lower than one for the higher frequency 30 GHz. As we would like an even lower number than 0.64 thermal photons at \( z = L \), so that the detection of the signal photons becomes easier, we try to further decrease it. The following values can be reached for \( f = 30 \) GHz:

\[
T_{\text{high}} = 300 \text{ K}, L = 0.01 \text{ m} \rightarrow N_{\text{thermal}}(z = L) = 0.25 \\
T_{\text{high}} = 100 \text{ K}, L = 0.01 \text{ m} \rightarrow N_{\text{thermal}}(z = L) = 0.08 \\
T_{\text{high}} = 15 \text{ K}, L = 1 \text{ m} \rightarrow N_{\text{thermal}}(z = L) = 0.15
\]

For the two lower frequencies and \( L = 1 \) m the number of thermal photons at the detector are even for 50 K too high to exactly detect a signal photon. For a length of \( L = 1 \) m the setup would have to be cooled to under 3 K to get comparable number of thermal photons for 1 and 6 GHz as at 15 K for 30 GHz. For the very short distance of 1 cm, the number of thermal photons at the detector is equal for all three frequencies. This equalization happens due to the short length. The transmission line is therefore not even in a state that is similar to an equilibrium. This leads to an overall low number of thermal photons as the number of thermal photons in the middle of the transmission line is less than 0.1 % bigger than at the detector.

The previous values for \( \alpha \) were extracted for \( \tan \delta = 0.005 \). We now look at the difference to the values for \( \tan \delta = 0.001 \). The attenuation constants are calculated similarly to (47):

\[
\alpha = -\frac{1}{\Delta z} \ln \left(10^{0.1 \alpha_d \Delta z}\right) \quad \text{with} \quad \alpha_d = \frac{-0.1 \text{dB cm}}{110 \text{GHz}} \cdot f
\]

With the same formulas as used before we get the following results:

<table>
<thead>
<tr>
<th>( f = \frac{\nu}{2\pi} ) [GHz]</th>
<th>( \frac{\alpha}{2\pi} )</th>
<th>( L ) [m]</th>
<th>( N_{\text{thermal}} )</th>
<th>( N_{\text{thermal}}(z = L) )</th>
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<tr>
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<td></td>
<td>10</td>
<td>4.14</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Values for different lengths \( L \), \( T_{\text{high}} = 300 \) K, \( \tan \delta = 0.001 \)

The number of thermal photons at the detector is much lower than for the attenuation constants for \( \tan \delta = 0.005 \). In the following table we will vary the temperature again.
### 3. Calculations and Results

\[ f = \frac{n}{2\pi} \text{ (GHz)} \]

\[ T_{\text{high}} \text{ [K]} \]

\[ N_{\text{thermal}} , N_{\text{thermal}}(z = L) \]

<table>
<thead>
<tr>
<th>( f )</th>
<th>( T_{\text{high}} )</th>
<th>( N_{\text{thermal}} )</th>
<th>( N_{\text{thermal}}(z = L) )</th>
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**Table 4:** Values for different temperatures \( T_{\text{high}}, L = 1 \text{ m} \), \( \tan \delta = 0.001 \)

The thermal photons at the detector are again much fewer than for \( \tan \delta = 0.005 \).

The reason for the noticeable difference between the values is that the ratio between the length and the attenuation constant is smaller. Hence the effect which appears in table 1, that the number of thermal photons at \( z = L \) becomes much lower than \( N_{\text{thermal}} \), can already be seen at longer distances.
4 Conclusion

Our expectation that the transfer of information with microwave photons is possible over distances of several metres or even several kilometres has not been approved. The transfer over such lengths is especially not possible at room temperature. At room temperature the detection of photons for $\tan \delta = 0.005$ could only be guaranteed over lengths of the magnitude of centimetres. For short lengths the assumption that the transmission line is in an equilibrium state becomes unlikely. But the description of the line as an equilibrium is required for the Nyquist theorem. Hence the description of the thermal photons becomes more difficult for short distances.

The transfer of photons over $L = 1$ m for $\tan \delta = 0.005$ can only be ensured for a frequency $f = 30$ GHz for temperatures of 15 K or lower. For the lower frequencies $f = 1$ GHz and $f = 6$ GHz the transmission line has to be even cooler than 3 K. For this low temperatures the contribution of the cold terminating parts is not negligible anymore.

The distances respectively temperatures with which the detection is possible are longer respectively higher for $\tan \delta = 0.001$. Therefore it is advantageous to get a very low $\tan \delta$. But even for $\tan \delta = 0.001$ the lengths and temperatures which are feasible are not at the stage that are needed for a proper information transfer.

Reviewing the paper I read before starting this thesis, one paper can be easily compared with our results. The paper about intracity quantum communication [2] discusses the transfer with microwaves too and investigates about the needed temperature on the transmission line which is the part of photon detection we had a look at in this thesis. In the paper it was stated that the transmission line has to be cooled to a few Kelvin. We tried to disprove this statement in conclusion but we can only approve the necessity of a cooled fiber. In the paper they use harmonic oscillators as a stage between the qubit and the channel to reduce the disturbance due to thermal photons. With the calculations we made there is no possibility to evaluate their usage. But it seems to be a good way to be able to increase the temperature respectively the length of the channel.

Because we did not look at the same properties of photon detection as it was done in the other papers that were mentioned in section 2.5, a comparison is not suitable.

As an overall conclusion it seems as if there are still a lot of aspects to develop before the transfer of quantum information becomes applicable in everyday life.
References


